Course name: **AP Calculus AB**

Revised: Aug 11, 2008

Course Description: In AP Calculus AB, students will explore four main ideas from calculus: limits, derivatives, indefinite integrals, and definite integrals. Problems will be approached through a balance of multiple representations including: graphically, numerically, analytically/algebraically, and verbally. Wherever practical, concepts will be applied to analyze real-world situations.

Graphing calculators and/or computers will be used on a regular basis to help solve problems, experiment, interpret results, and support conclusions.

Students will learn to communicate mathematics through the use of a math journal and having time each day to work in groups to exchange ideas and approaches and to reflect on homework assignments.

A variety of assessments will be used to including paper based tests and quizzes, homework, group and individual laboratory work. In order to prepare students for the AP exam, tests and quizzes will contain questions similar to those that appear on the AP exam in terms of content, difficulty, and structure.

Primary text(s) and other major resources:
Calculus of a single variable Larson, Hostetler, and Edwards
(D.C. Heath and Company, 1994)

○ **unit number & title**	**Objectives** (specific skills and knowledge students will have)	**Essential Concepts**	**Assessment** Students must show proficiency with MLR skills and knowledge in assessments marked Essential in order to progress to the next course level.
U1 Prerequisites and Analysis of Graphs	Objective set 1: 2-3 Weeks Student will: • review—the real number system • review—the cartesian plane • review—graphs of equations • review—lines in the plane • review—functions • review—trigonometric functions	How are real numbers and sets of real numbers be represented, classified, and ordered? What algebraic techniques are commonly used in calculus? How can data and relationships be organized and represented graphically?	• **Homework and quizzes will be given on a regular basis throughout the course** Project #1 Test #1

U2
Limits and Continuity
	Objective set:
	4-5 weeks
	
	Students will:
	• Evaluate Limits at a Point
	  ○ 1 sided limits
	  ○ 2 sided limits
	  ○ Sandwich Theorem
	• Evaluate Limits involving Infinity
	  ○ Asymptotic Behavior
	  ○ End Behavior models
	  ○ Properties of Limits (Algebraic Analysis)
	  ○ Visualizing Limits (Graphic Analysis
	• Determine Continuity of a function
	  ○ Continuity at a Point
	  ○ Continuous Functions
	  ○ Discontinuous Functions
	    ▪ Removable Discontinuity
	    ▪ Jump Discontinuity
	    ▪ Infinite Discontinuity
	• Determine and Analyze Rates of Change and Tangent Lines
	  ○ Average rate of change
	  ○ Tangent line to a curve
	  ○ Slope of a curve (algebraically and graphically)
		What is a limit?
		How do you find a limit with a table, graph, or analytically?
		When does a limit not exist?
		What is the definition of continuity for a function on an open or closed interval?
		
		What is a continuous function?
		How can I determine if a function is continuous?
		
		What is meant by rates of change?
		How is the rate of change determined?
		How are tangent lines and a rate of change related?
		What is a normal line and how can I determine the formula for a normal	

<table>
<tr><th>○ unit number & title</th><th>Objectives
(specific skills and knowledge students will have)</th><th>Essential Concepts</th><th>Assessment
Students must show proficiency with MLR skills and knowledge in assessments marked Essential in order to progress to the next course level.</th></tr>
<tr><td></td><td>○ Normal line to a curve (algebraically and graphically)
○ Instantaneous rate of change</td><td>line?</td><td>Project #2
Test #2</td></tr>
<tr><td>U3
The Derivative</td><td>Objective Set
5-6 Weeks

Students will:
• Determine Rates of Change
  ○ Average Speed
  ○ Instantaneous Speed
• Use a variety of methods to determine the Derivative of a Function
  ○ Definition of the derivative (difference quotient)
  ○ Derivative at a Point
  ○ Relationships between the graphs of f and f'
  ○ Graphing a derivative from data
  ○ One sided derivatives
• Determine if a function is differentiable
  ○ Cases where f'(x) might fail to exist
  ○ Local linearity
  ○ Derivatives on the calculator (Numerical derivatives using NDERIV)
  ○ Symmetric difference quotient
  ○ Relationship between differentiability and continuity
  ○ Intermediate Value Theorem for Derivatives
• Use Rules for Differentiation to determine derivatives of functions
  ○ Constant, Power, Sum, Difference, Product, Quotient Rules
  ○ Higher order derivatives
• Apply concepts of the derivative to analyze motion problems
  ○ Position, velocity, acceleration, and</td><td>How do I determine a rate of change?
What is the difference between an average rate of change and an instantaneous rate of change?
How I determine the derivative of a function?

How do I determine if a function is differentiable?
How do I determine if a function is continuous?
What facts can we conclude about a continuous function?

How can we use derivatives to analyze motion?</td><td></td></tr>
</table>

○ **unit number & title**	**Objectives** (specific skills and knowledge students will have)	**Essential Concepts**	**Assessment** Students must show proficiency with MLR skills and knowledge in assessments marked Essential in order to progress to the next course level.
	jerk ○ Particle motion ○ L'HÔpital's Rule • Apply concepts of the derivative to analyze economic problems ○ Marginal Cost ○ Marginal Revenue ○ Marginal Profit • Determine derivatives of Trigonometric Functions • Use the Chain Rule to derive composite functions • Determine rate of change using Implicit Differentiation ○ Differential Method ○ Y' Method • Determine derivatives of Inverse Trigonometric Functions • Determine derivatives of Exponential and Logarithmic Functions	How can we use derivatives to analyze economic problems? How do we determine the derivative of trigonometric functions? What are composite functions and how do we determine the derivative? What are implicit functions and how do we determine the derivative? What are inverse trigonometric functions and how do we determine the derivative? What are exponential functions and logarithmic functions and how do we determine the derivative	Project #3 Test #3
U4 Applications of the Derivative	Objective Set 5-6 Weeks Students will: • Locate Extreme Values of a function ○ Relative Extrema ○ Absolute Extrema ○ Extreme Value Theorem ○ Definition of a Critical Point • Interpret Implications of the Derivative ○ Rolle's Theorem ○ Mean Value Theorem ○ Increasing and Decreasing functions • Produce accurate graphs by using the relationships of f' and f'' with f(x)	What are extrema and where do they occur? What can we interpret about a function from the derivative? How is a function related to its first	

○ **unit number & title**	**Objectives** (specific skills and knowledge students will have)	**Essential Concepts**	**Assessment** Students must show proficiency with MLR skills and knowledge in assessments marked Essential in order to progress to the next course level.
	○ First derivative test for relative max/min ○ Second Derivative ▪ Concavity ▪ Inflection Points ▪ Test for relative max/min • Analyze optimization problems using calculus • Estimate functions using Linearization models ○ Local Linearization ○ Tangent Line approximation ○ Differentials • Analyze Related Rate problems	and second derivatives? What is an optimization problem and how can we use derivatives to solve them? How can we produce estimates of the value of a curve using linearization models? What is a related rate problem and how can we use derivatives to solve them?	Project #4 – Saving material by improving the design of cereal boxes Test #4
U5 The Definite Integral	Objective Set 3-4 Weeks Students will: • Approximate Areas between curves using summations ○ Riemann sums ▪ Left ▪ Right ▪ Midpoint ▪ Trapezoidal • Compare and contrast definite integrals to Riemann sums • Use Properties of Definite Integrals ○ Power Rule ○ Mean Value Theorem for Definite Integrals • Understand how The Fundamental Theorem of Calculus describes the inverse relationship between the integral and derivative	What methods can be used to approximate areas underneath curves? How can we modify our approximation methods to improve accuracy? What are the basic properties of definite integrals? How are derivatives and integrals related?	Project #5 Test #5

○ **unit number & title**	**Objectives** (specific skills and knowledge students will have)	**Essential Concepts**	**Assessment** Students must show proficiency with MLR skills and knowledge in assessments marked Essential in order to progress to the next course level.
	○ Part 1 ○ Part 2		
U6 Differential Equations and Mathematical Modeling	Objective Set 3 Weeks Students will: • Produce Slope Fields for differential equations • Evaluate Antiderivatives using common formulas ○ Indefinite Integrals ○ Power Formulas ○ Trigonometric Formulas ○ Exponential and Logarithmic formulas • Analyze Logistic Growth models	What is a differential equation? How can we visualize the solutions for a differential equation? What are some common antiderivatives? What are logistic growth models and how do I analyze them using calculus?	Project #6 Test #6
Unit 7 Applications of Definite Integrals	Objective Set 3-4 Weeks Students will explore the following topics: • Evaluate and interpret Integrals as net change ○ Calculating distance traveled ○ Consumption over time ○ Net Change from data • Calculate areas between curves ○ Integrating with respect to x ○ Integrating with respect to y • Calculate areas between intersecting curves ○ Integrating with respect to x ○ Integrating with respect to y • Calculate Volume of solids ○ Cross sections ○ Disc Method ○ Shell Method	How can integrals be used to describe motion or other changing variables? How can we use integrals to calculate areas between curves? How can we use integrals to calculate volumes of solids?	Project #7 – Determine distance of a trip using speed and time data. Test #7

Notes

This is a sample of a sheet that we typically use to analyze various calculus problems using a multiple approaches. We use it to compliment our work with the graphing calculators. The "notes" section is used for a written and analytical approach. The "table" section below is used for numerical approach with enough room to examine the function, first derivative, and second derivative. The "graphs" section is for a graphical approach allowing the student to see function and derivative behavior either on the coordinate plane and/or using intervals along a number line.

We use this sheet for selected homework problems or in-class problems along with our Ti graphing calculators

Line Graphs

Function

1st Derivative

2nd Derivative

Graph of Function

Graph of 1st Derivative

Graph of 2nd Derivative

Calculus Project
Designing an Environmentally Friendly Package

Introduction

You have been hired by the CEO of a major company to design the packaging for its newest cereal. The CEO takes environmental issues very seriously and wants to make sure that the box that you design uses as little material as possible.

Objectives:

- You will be able to calculate the derivative of a function using the product rule
- You will be able to calculate the derivative of a function using the quotient rule
- You will be able utilize derivatives to analyze optimization problems.
- You will be able to show multiple representations of a mathematical solution (algebraic/analytical, numeric, graphical, written english)

Specifications for the packaging

- The cereal must be packaged in a rectangular box.
- The base must be ______ times the width. (number to be assigned by teacher)
- The volume of the box must hold exactly _____ inches^3. (number to be assigned by teacher)
- The box must use the least amount of cardboard (minimize the surface area) to save money.
- You must write a clear step by step description of your design process supported by functions, derivatives, graphs, and data tables to convince the CEO that your design is the most efficient use of material.

Teacher Notes:
This Box Project is an example of one of our project that requires the student to apply calculus to solve a real world problem. In this case, the student has to design a cereal box that meets the volume requirements while minimizing the amount of material used in order to help minimize the environmental impact. The students must use an analytical, graphical, numerical, and written approach to describe their solution. Furthermore, to complete the real-world hands-on experience, the students must actually construct their proposed box.

The next project (Zoo Project) is also a real-world application of calculus. Students use limited materials to construct a zoo that maximizes living spaces for the animals. Again, the students use multiple approaches (analytical, written, numerical, and graphical) to help gain a deeper understanding of the solution. Consistent with the hands-on experience, students complete the project with a blueprint scaled map of their proposed zoo configuration.

Students will explore these, and several other application of calculus throughout the year.

Calculus

Project 1: Design an environmentally friendly package

	4	3	2	1
Name, Date, Class, and Title	All of Name, Date, Class, and Project title are clearly written at top of the paper.	Most of Name, Date, Class, and Project title are written at top of the paper.	Some of Name, Date, Class, and Project title are written at top of the paper.	None of Name, Date, Class, and Project title are written at top of the paper.
Written Description of your process	A very clear step by step description of your design strategy is included. All major steps are explained clearly and precisely. Key mathematical terms are used appropriately.	A mostly clear step by step description of your design strategy is included. Most, but not all, major steps are explained clearly and precisely. Key mathematical terms are used appropriately.	A step by step description of your design strategy is included. Some, but not all, major steps are explained. Little or improper use of mathematical terms.	A step by step description of your design strategy is not included or lacks most major steps. Little or no use of mathematical terms.
Algebraic Representation of your process	A very clear algebraic solution to your design strategy is included. This means that proper formulas and functions are correctly written and solved. All steps are neatly shown until a conclusion is found. No mistakes.	A mostly clear algebraic solution to your design strategy is included. This means that proper formulas and equations are correctly written and solved. Most, but not all, steps are neatly shown until a conclusion is found. Only one or two mistakes.	An algebraic solution to your design strategy is included. This means that proper formulas and equations are correctly written and solved. Some steps are shown, but some are skipped making the process difficult to follow. Three or four mistakes.	An algebraic solution to your design strategy is not included or has many mistakes. Very few or no steps are shown making the process difficult to follow. More than four mistakes.
Numerical Representation of your process (Data Table of functions and derivatives)	A very clear data table shows the behavior of all key functions and derivatives that support your design strategy. All of the most important data values are highlighted and their significance to your solution is accurately described. Columns are labeled with correct units.	A mostly clear data table shows the behavior of all key functions and derivatives that support your design strategy. Some of the most important data values are highlighted and their significance to your solution is described with few mistakes. Columns are labeled with correct units.	A data table shows the behavior of some, but not all, key functions and derivatives. Few or none of the most important data values are highlighted. Incorrect or no use of labels and units.	No data table included

	4	3	2	1
Graphical Representation of your process	Very clear graphs show the behavior of all key functions and derivatives that support your design strategy. All of the important points on the curves are highlighted and their significance to your solution is clearly described. All axis are properly labeled and have correct units.	Graphs show the behavior of all or most key functions and derivatives that support your design strategy. Most of the important points on the curves are highlighted and their significance to your solution is described with few mistakes. Most axis are properly labeled and have correct units.	Graphs show the behavior of some functions and derivatives. Few of the important points on the curves are highlighted and their significance to your solution is not made clear. Axis are not properly labeled.	Graphs are not included or have numerous major mistakes.
Use of calculus concepts	Appropriate and accurate application and interpretation of derivatives is an important part of your design solution. These concepts are clearly incorporated in your written, algebraic, numeric, and graphical representations.	Appropriate and accurate application and interpretation of derivatives is part of your design solution. These concepts are incorporated in your written, algebraic, numeric, and graphical representations with some mistakes.	Appropriate and accurate application and interpretation of derivatives is part of your design solution. These concepts are incorporated in your some of your written, algebraic, numeric, and graphical representations with many mistakes.	Appropriate and accurate application and interpretation of derivatives is not part of your design solution or contains many mistakes.
Actual Cereal Box	You have constructed the full size cereal box that is accurate to your calculations. All key dimensions of the box are clearly labeled with units (length, width, height, volume and surface area)	You have constructed full size cereal box that is mostly accurate to your calculations with one or two mistakes. Most key dimensions of the box are labeled with units (length, width, height, volume and surface area)	You have constructed the cereal box that is somewhat accurate to your calculations with three or four mistakes. Some key dimensions of the box are labeled with units (length, width, height, volume and surface area)	You have not constructed the cereal box or you have more than four mistakes.
On Time	The completed project is delivered on or before the due date: **Start of class on Tuesday December 9.**	The completed project is delivered within one day after the due date: **Start of class on Tuesday December 9.**	The completed project is delivered within three days after the due date: **Start of class on Tuesday December 9.**	The completed project is delivered more than three days after the due date: **Start of class on Tuesday December 9**

Calculus

Project – Optimize our Zoo Using Functions (Revised)

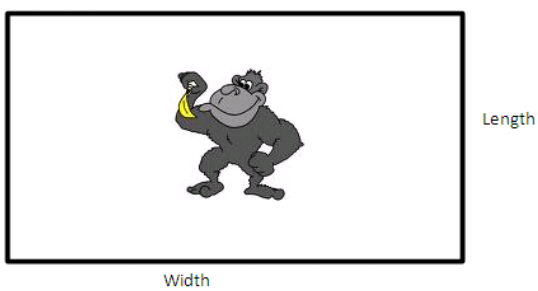
Introduction

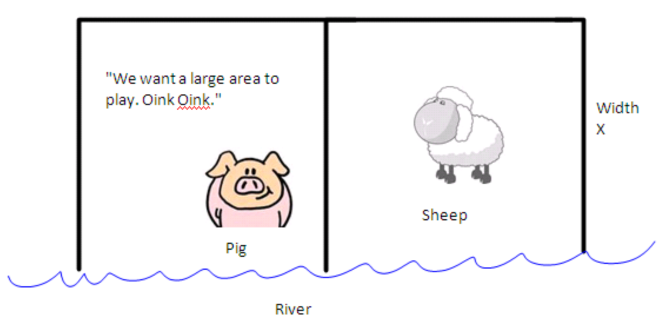
You have been hired by the local community to advise them on how to best design their new zoo. They have limited materials to use and they want to ensure that they get the most out of the materials. You will need to examine their requirements, determine the best way to meet the requirements, and make a scale model or map of your proposed design.

Objectives:

- You will be able to model real situations with functions.
- You will be able to represent the functions algebraically, numerically (data table), and graphically.
- You will be able to use proportions to produce a map to scale.

Zoo Requirements

1. **Gorilla**
   a. We have __________ feet of fence to use for the gorilla cage.
   b. We want the gorilla to live in a rectangular space with the maximum area possible to swing around.
   c. 

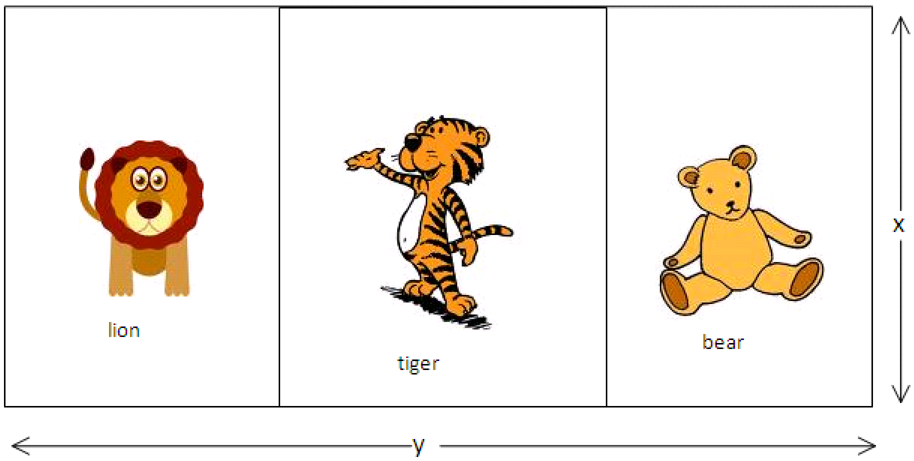
2. **Sheep and Pigs**
   a. We have ______________ feet of fence to use for the sheep and pigs.
   b. We want the sheep and pigs to live in two adjacent rectangular pens.
   c. One side of the sheep and pigs area will be bordered by a river for drinking
   d. We want to give the sheep and pigs the maximum area possible.
   e. The pig area should be the same size as the sheep area.
   f. 

3. Lions, Tigers, and Bears

a. We have _____________ feet of fence to use for the lions tigers and bears.

b. The animals will live in one rectangular area divided into three equal parts as shown below.

c. We want the lions, tigers, and bears to live in the maximum area possible.

d. 

4. Shark Aquarium

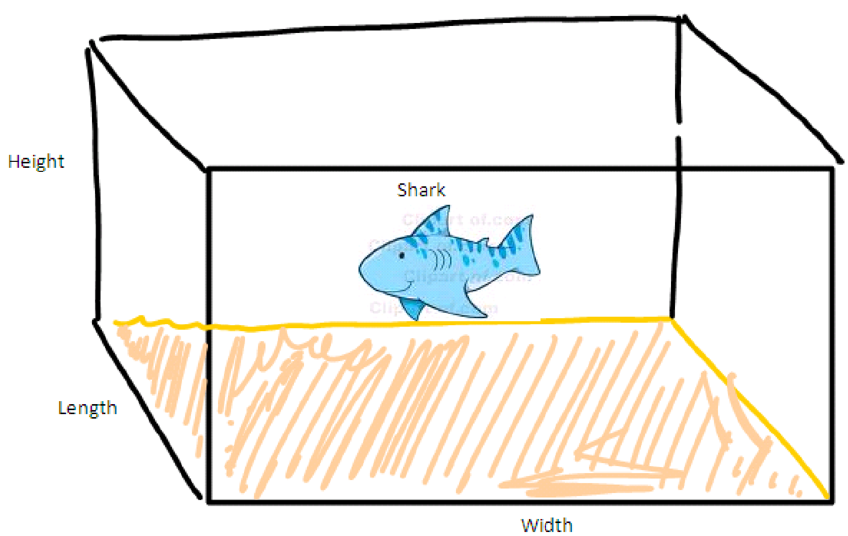
a. We want a rectangular aquarium with the left, right, front, back, and bottom is made of clear glass so people can see the sharks

b. The top is open (no glass used for top)

c. The length should be ______________ times the width.

d. The tank should hold a volume of _____________.

e. We want to minimize the amount of glass that we use because the material is very expensive.

f. 

Element	4 - Mastered	3 - Proficient	2 - Developing	1 -Emerging
Labeled picture of description	You always do these with no errors: ☐ draw a clear picture of each application clearly labeled with all known and unknown variables.	You do most of these with few (1-2) minor errors: ☐ Draw a clear picture of each application clearly labeled with all known and unknown variables.	You do most of these with some (3-6) errors ☐ Draw a clear picture of each application clearly labeled with all known and unknown variables.	You do few or none of these or have major errors. ☐ Draw a clear picture of each application clearly labeled with all known and unknown variables.
Use Algebra to create the Appropriate Function(s)	You always do these no errors: ☐ write key equations/ functions that show how each of the variables are related to one another . (i.e. V=lwh). ☐ arrange these equations to write an appropriate function(s) that can be used to describe each problem (i.e. Surface Area function)	You do most of these with few (1-2) minor errors: ☐ write key equations/ functions that show how each of the variables are related to one another . (i.e. V=lwh). ☐ arrange these equations to write an appropriate function(s) that can be used to describe each problem (i.e. Surface Area function)	You do most of these with some (3-6) errors ☐ write key equations/ functions that show how each of the variables are related to one another . (i.e. V=lwh). ☐ arrange these equations to write an appropriate function(s) that can be used to describe each problem (i.e. Surface Area function)	You do few or none of these or have major errors. ☐ write key equations/ functions that show how each of the variables are related to one another . (i.e. V=lwh). ☐ arrange these equations to write an appropriate function(s) that can be used to describe each problem (i.e. Surface Area function)
Numerical Behavior of the function	You always do these no errors: ☐ produce a clear table(s) of values that show the behavior of your function(s) ☐ Use labels, variables, and units on your table(s) so it's very easy to know what the data means. ☐ Highlight any data on the table(s) important to your solution and clearly explain why it helps you solve the problem.	You do most of these with few (1-2) minor errors: ☐ produce a clear table(s) of values that show the behavior of your function(s) ☐ Use labels, variables, and units on your table(s) so it's very easy to know what the data means. ☐ Highlight any data on the table(s) important to your solution and clearly explain why it helps you solve the problem.	You do most of these with some (3-6) errors ☐ produce a clear table(s) of values that show the behavior of your function(s) ☐ Use labels, variables, and units on your table(s) so it's very easy to know what the data means. ☐ Highlight any data on the table(s) important to your solution and clearly explain why it helps you solve the problem.	You do few or none of these or have major errors. ☐ produce a clear table(s) of values that show the behavior of your function(s) ☐ Use labels, variables, and units on your table(s) so it's very easy to know what the data means. ☐ Highlight any data on the table(s) important to your solution and clearly explain why it helps you solve the problem.
Graphical Behavior of the Function	You always do these with no errors: ☐ produce a clear graph(s) that show the behavior of your function(s) ☐ Use labels, variables, and units on your graph(s) so it's very easy to know what the curve means ☐ Highlight any point(s) on the curve that is important to your solution and clearly explain why it helps you solve the problem.	You do most of these with few (1-2) minor errors: ☐ produce a clear graph(s) that show the behavior of your function(s) ☐ Use labels, variables, and units on your graph(s) so it's very easy to know what the curve means ☐ Highlight any point(s) on the curve that is important to your solution and clearly explain why it helps you solve the problem.	You do most of these with some (3-6) errors ☐ produce a clear graph(s) that show the behavior of your function(s) ☐ Use labels, variables, and units on your graph(s) so it's very easy to know what the curve means ☐ Highlight any point(s) on the curve that is important to your solution and clearly explain why it helps you solve the problem.	You do few or none of these or have major errors. ☐ produce a clear graph(s) that show the behavior of your function(s) ☐ Use labels, variables, and units on your graph(s) so it's very easy to know what the curve means ☐ Highlight any point(s) on the curve that is important to your solution and clearly explain why it helps you solve the problem.

Element	4 - Mastered	3 - Proficient	2 - Developing	1 -Emerging
Solve the problem	You always do these with no errors: ☐ Clearly interpret all the information from your graphs, tables, and algebra to solve the problem. ☐ The solution could be one number or a set of numbers so labels and units are used to clarify. ☐ If possible, a picture is used to clarify your solution and show that you understand how the numbers solve the original problem. ☐ Check to see that your solution makes sense.	You do most of these with few (1-2) minor errors: ☐ Clearly interpret all the information from your graphs, tables, and algebra to solve the problem. ☐ The solution could be one number or a set of numbers so labels and units are used to clarify. ☐ If possible, a picture is used to clarify your solution and show that you understand how the numbers solve the original problem. ☐ Check to see that your solution makes sense.	You do most of these with some (3-6) errors ☐ Clearly interpret all the information from your graphs, tables, and algebra to solve the problem. ☐ The solution could be one number or a set of numbers so labels and units are used to clarify. ☐ If possible, a picture is used to clarify your solution and show that you understand how the numbers solve the original problem. ☐ Check to see that your solution makes sense.	You do few or none of these or have major errors. ☐ Clearly interpret all the information from your graphs, tables, and algebra to solve the problem. ☐ The solution could be one number or a set of numbers so labels and units are used to clarify. ☐ If possible, a picture is used to clarify your solution and show that you understand how the numbers solve the original problem. ☐ Check to see that your solution makes sense.
Scale Map	You always do these with no errors: ☐ Make a creative, accurately scaled map that represents your proposal for the configuration of the zoo. ☐ Show a legend that indicates the scale of actual lengths to map lengths. ☐ Label all required areas and clearly indicate any key measurements	You do most of these with few (1-2) minor errors: ☐ Make a creative, accurately scaled map that represents your proposal for the configuration of the zoo. ☐ Show a legend that indicates the scale of actual lengths to map lengths. ☐ Label all required areas and clearly indicate any key measurements	You do most of these with some (3-6) errors ☐ Make a creative, accurately scaled map that represents your proposal for the configuration of the zoo. ☐ Show a legend that indicates the scale of actual lengths to map lengths. ☐ Label all required areas and clearly indicate any key measurements	You do few or none of these or have major errors. ☐ Make a creative, accurately scaled map that represents your proposal for the configuration of the zoo. ☐ Show a legend that indicates the scale of actual lengths to map lengths. ☐ Label all required areas and clearly indicate any key measurements
On Time	You're project is completed and handed in before or at the beginning of class on the due date: **January 14, 2009**	You're project is completed and handed in within 2 days past the due date: **January 14, 2009**	You're project is completed and handed in within 4 days past the due date: **January 14, 2009**	You're project is completed and handed in more than 4 days past the due date: **January 14, 2009**