



# LEE ACADEMY

Lee, Maine USA

*Official Curriculum*

## Pre-Calculus

Rev. 2011

*Course description:* In *Precalculus*, students utilize their understanding of arithmetic, algebra, and geometry to solve situations involving polynomial, rational, exponential, and trigonometric functions, conic sections, and partial fractions. Assessments are done in a variety of formats: multiple-choice, short response, extended response, and major situation-based applications.

As a pilot project, learning goals for this course are derived from the *Common Core*, newly evolving national standards in the United States. Additional information may be found at: <http://www.corestandards.org/the-standards/mathematics>

Primary text(s) and other major resources

*Precalculus: Graphical, Numerical, Algebraic*—6<sup>th</sup> edition, Demana, Waits, Foley and Kennedy  
Pearson, Addison-Wesley—2004

Unit Length and Common Core Standards	Objectives	Essential Questions	Assessments
<p><b>Unit 1</b> Review, mathematical communication, and understanding quantities</p> <p>Goal Set 1:</p> <p><b>M1:</b> Students will develop a framework for approaching, formulating, computing, interpreting, and reporting problems and their solutions.</p> <p><b>N-Q1:</b> Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret scale and the origin in graphs and data displays.</p> <p><b>N-Q2:</b> Define appropriate quantities for the purpose of descriptive modeling.</p> <p><b>N-Q3:</b> Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.</p>	<ol style="list-style-type: none"> <li>1. Students will identify the steps in the problem solving process following the procedure: problem, formulate, compute, interpret, validate, and report.</li> <li>2. Students will apply the process to real world problems.</li> <li>3. Students will solve equations graphically, numerically, algebraically, and report their results verbally and in writing.</li> <li>4. Students will solve inequalities algebraically and graphically, and report the meaning of the solutions verbally and in writing.</li> <li>5. Students will be able to define units in a general context.</li> <li>6. Students will be able to identify common units for physical quantities.</li> <li>7. Students will be able to identify when quantities can be added or subtracted based on units, as well as what quantities can be multiplied to get the desired units.</li> <li>8. Students will be able to translate from one type of similar unit to another (for example, feet into meters or vice versa).</li> <li>9. Students will choose the best quantity for modeling a given scenario.</li> <li>10. Students will be able to select the best scale to view the problem and its solution both numerically and graphically.</li> </ol>	<ol style="list-style-type: none"> <li>1. How can we write explanations of mathematical problems and solutions using words?</li> <li>2. What is a mathematical model, and how can we construct them so they may be applied to solve real world problems?</li> <li>3. What different information can be determined about a problem when we examine it from a graphical, numerical, and algebraic standpoint?</li> <li>4. In what circumstances in the real world would inequalities be both appropriate and an accurate display of what we want to accomplish?</li> <li>5. What role do units play in the construction of mathematical models?</li> <li>6. What information can we get about a situation described in a mathematical model based entirely on the units used?</li> <li>7. How should the selection of scale be different between different mathematical models? (ex: examining cells in biology and building houses?)</li> </ol>	<p><i>(Note: this unit is mostly covered in the prerequisite chapter and chapter 1.1, with the exception of quantity and units, which must be addressed separately.)</i></p>

<p><b>Unit 2</b> Functions</p> <p><b>F-IF1:</b> Understand that a function from one set (called the domain) to another set (called the range) assigns each element of the domain exactly one element of the range. If <math>f</math> is a function and <math>x</math> is an element of its domain, then <math>f(x)</math> denotes the output of <math>f</math> corresponding to the input <math>x</math>. The graph of <math>f</math> is the graph of the equation <math>y = f(x)</math>.</p> <p><b>F-IF2:</b> Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.</p> <p><b>F-IF4:</b> For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</p> <p><b>F-IF5:</b> Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function <math>h(n)</math> gives the number of person-hours it takes to assemble <math>n</math> engines in a factory, then the positive integers would be an appropriate domain for the function.</p> <p><b>F-IF6:</b> Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.</p>	<ol style="list-style-type: none"> <li>1. Students will be able to recognize a function as a mathematical machine, where a quantity input is transformed into a quantity output.</li> <li>2. Students will be able to define the domain and range of a function based on the algebraic expression, graph, data table, or list of elements and outputs.</li> <li>3. Students will be able to judge whether something is or is not a function based on the one output for every input rule.</li> <li>4. Students will be able to use graphs or numerical tables to determine when a function is increasing, decreasing, or constant.</li> <li>5. Students will be able to determine whether the range of a function as an upper or lower bound.</li> <li>6. Students will be able to identify and compute the local and absolute maximums and minimums.</li> <li>7. Students will be able to define and identify vertical and horizontal asymptotes, and determine if either is present in the domain of the function.</li> <li>8. Students will be able to identify graphically the twelve basic functions, and express key features of each based on the graph.</li> <li>9. Students will be able to calculate the average rate of change of a function, and interpret the meaning of average rate of change in the context of the problem.</li> <li>10. Students will be able to construct appropriate functions to model real problems.</li> <li>11. Students will be able to manipulate and combine functions into other functions, and interpret the meaning of the new function.</li> <li>12. Students will be able to define and construct inverse functions.</li> <li>13. Students will be able to identify when two functions are inverses</li> </ol>	<ol style="list-style-type: none"> <li>1. What would an appropriate analogy for a function be?</li> <li>2. Why is a function restricted to one output for every input?</li> <li>3. What are the forms that a function can take, and what are the advantages of each form?</li> <li>4. What information can we determine from the graph of a function?</li> <li>5. What are some examples of things that behave like functions from real life?</li> <li>6. What can we learn about a situation that has vertical or horizontal asymptotes?</li> <li>7. In what real world situations do we use average rate of change?</li> <li>8. How can two functions be combined?</li> <li>9. In what circumstances could we describe a situation as the combination of two functions?</li> </ol>	<p><i>(Note: This is a new section based on the national Common Core and these topics are represented on the SAT's and the SAT II's.)</i></p>
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<p><b>F-IF9:</b> Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.</p> <p><b>F-BF1:</b> Write a function that describes a relationship between two quantities.</p> <p>a. Determine an explicit expression, a recursive process, or steps for calculation from a context.</p> <p>b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.</p> <p>c. (+) Compose functions. For example, if <math>T(y)</math> is the temperature in the atmosphere as a function of height, and <math>h(t)</math> is the height of a weather balloon as a function of time, then <math>T(h(t))</math> is the temperature at the location of the weather balloon as a function of time.</p> <p><b>F-BF3:</b> Identify the effect on the graph of replacing <math>f(x)</math> by <math>f(x) + k</math>, <math>kf(x)</math>, <math>f(kx)</math> and <math>f(x + k)</math> for specific values of <math>k</math> (both positive and negative); find the value of <math>k</math> given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.</p> <p><b>F-BF4:</b> Find inverse functions.</p> <p>a. Solve an equation of the form <math>f(x) = c</math> for a simple function <math>f</math> that has an inverse and write an expression for the inverse.</p>	<p>based on graphs and numerical tables.</p> <p>14. Students will be able to identify what shifts and transformations will look like graphically when they are applied to a function.</p> <p>15. Students will model real world problems using functions, and communicate the solution graphically, numerically, and verbally.</p>		
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<p>b. (+) verify by composition that one function is the inverse of another.</p> <p>c. (+) read values of an inverse function from a graph or a table.</p> <p>d. (+) Produce an invertible function from a non-invertible function by restricting domain.</p>			
<p><b>Unit 3</b> Polynomials, power models, and rational functions</p> <p><b>F-IF7:</b> Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.</p> <p>a. Graph linear and quadratic functions and show intercepts, maxima, and minima.</p> <p>b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.</p> <p>c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.</p> <p>d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.</p> <p><b>N-RN1:</b> Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.</p> <p><b>F-IF8:</b> Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.</p> <p>a. Use the process of factoring and completing the square in a quadratic</p>	<ol style="list-style-type: none"> <li>Students will construct graphs of linear and quadratic functions, identifying concepts such as increasing or decreasing, positive or negative, and extrema.</li> <li>Students will be able to use completing the square to algebraically find the zeros and extrema of a quadratic function.</li> <li>Students will use polynomial functions to model real world problems numerically, graphically, and algebraically.</li> <li>Students will solve polynomial functions for zeros using polynomial long division or synthetic division.</li> <li>Students will define an imaginary number and a complex number.</li> <li>Students will be able to apply algebraic principles to complex numbers.</li> <li>Students will be able to identify the complex conjugate of a complex number.</li> <li>Students will construct a graph of a complex number on a rectangular coordinate axis.</li> <li>Students will be able to apply the fundamental theorem of algebra to find the non-real zeros of a polynomial function.</li> </ol>	<ol style="list-style-type: none"> <li>What information does a graph of a polynomial function have that may not be present in the algebraic model?</li> <li>What situations in real life can be modeled by a quadratic or polynomial equation?</li> <li>How can you determine the zeros of a polynomial equation?</li> <li>What is the purpose of imaginary numbers?</li> </ol>	<p>--- Linear, Quadratic, and Polynomial Equations REMIX exam</p>

<p>function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.</p> <p>b. Use the properties of exponents to interpret expressions for exponential functions.</p> <p><b>N-CN1:</b> Know there is a complex number <math>i</math> such that <math>i^2 = -1</math>, and every complex number has the form <math>a + bi</math> with <math>a</math> and <math>b</math> real.</p> <p><b>N-CN2:</b> Use the relation <math>i^2 = -1</math> and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.</p> <p><b>N-CN3:</b> Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.</p> <p><b>N-CN4:</b> Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.</p>			
<p><b>Unit 4</b> Exponential, logistic, and logarithmic functions</p> <p><b>F-IF7e:</b> Graph exponential and logarithmic functions, showing intercepts and end behavior.</p> <p><b>F-BF5:</b> Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.</p> <p><b>F-LE1c:</b> Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to</p>	<ol style="list-style-type: none"> <li>1. Students will be able to identify an exponential function from the form of the equation.</li> <li>2. Students will construct and interpret graphs of exponential functions.</li> <li>3. Students will be able to determine the end behavior of an exponential function, and interpret the meaning of the end behavior in the context of real world situations.</li> <li>4. Students will model population growth and decay, and interpret the computed information in context.</li> <li>5. Students will be able to define a logarithmic equation, and interpret the meaning of the logarithm.</li> <li>6. Students will be able to construct</li> </ol>	<ol style="list-style-type: none"> <li>1. What does an exponential function represent?</li> <li>2. What are the consequences of unbounded exponential growth?</li> <li>3. What are the characteristics of a graph of an exponential function?</li> <li>4. In what elements of life might we see exponential decay?</li> <li>5. How are exponential and logarithmic functions related?</li> </ol>	<p><u>Logarithms and Exponential Functions Exam</u> — this exam also serves as a sample Pre-AP-style test</p> <p><u>Music Mixing Boards and Logarithms</u> handout—used after experiencing the procedures for mixing sound with a board &amp; public address (PA) system.</p> <p><u>Application of Trigonometry Exploration Project</u></p>

<p>another.</p> <p><b>F-LE2:</b> Construct exponential functions given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).</p> <p><b>F-LE3:</b> Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.</p> <p><b>F-LE4</b> For exponential models, express as a logarithm the solution to <math>ab^{ct} = d</math> where <math>a</math>, <math>c</math>, and <math>d</math> are numbers, and the base <math>b</math> is 2, 10, or <math>e</math>; evaluate the logarithm using technology.</p> <p><b>F-LE5:</b> Interpret the parameters in a linear or exponential function in terms of a context.</p>	<p>graphs of logarithms, or if shown a graph, students will be able to identify the graph as logarithmic based on key features such as asymptotes and intercepts.</p> <p>7. Students will be able to solve logarithmic and exponential functions in order to determine key values for real-world problems.</p>	<p>6. How can we model a real life situation with a logarithmic equation?</p> <p>7. How does sound intensity relate to logarithms?</p> <p>8. When is it more accurate to use a logistic equation?</p>	
<p><b>Unit 5</b> Trigonometry</p> <p>F-TF Extend the domain of trigonometric functions using the unit circle</p> <p>1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.</p> <p>2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.</p> <p>3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for <math>\pi/3</math>, <math>\pi/4</math> and <math>\pi/6</math>, and use the unit circle to express the values of sine, cosines, and tangent for <math>x</math>, <math>\pi+x</math>, and <math>2\pi-x</math> in terms of their values for <math>x</math>, where <math>x</math> is any real number.</p> <p>4. (+) Use the unit circle to explain symmetry (odd</p>	<p>1. Students will be able to convert between radians and degrees.</p> <p>2. Students will be able to identify the advantages and disadvantages of each system, and be able to explain why radians are important to use in most situations.</p> <p>3. Students will extend their right triangle model to include the unit circle, and will be able to use the unit circle to gather key information about an equation.</p> <p>4. Students will recognize the cyclical nature of the unit circle, and as such they will be able to define periodicity, determine whether a function is odd or even, and be able to handle angle measurements of any size.</p> <p>5. Students will recognize what periodic events in their daily lives can be modeled with trig functions, and will be able to determine when a trig function is appropriate.</p>	<p>1. What is the difference between radians and degrees, and when is it a better choice to use radians?</p> <p>2. How can we interpret the meaning of sine and cosine in words?</p> <p>3. How can right triangle trigonometry be used in construction?</p> <p>4. When we extend the model of trigonometry to the full unit circle, how can we interpret the negative numbers?</p> <p>5. How can temperature variation across seasons be modeled using either sine or cosine?</p> <p>6. What is the meaning of periodicity, and how could it influence our</p>	<p><u>Basic Trigonometry and Graphing Exam</u> — a pre-AP-style test</p>

<p>and even) and periodicity of trigonometric functions.</p> <p>Model periodic phenomena with trigonometric functions</p> <p>5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. ★</p> <p>6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.</p> <p>7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. ★</p>	<p>6. Students will be able to find the angle of elevation when they are presented a value of a trig function.</p> <p>7. Students will be able to graph and model trig functions on calculators and computers, and draw conclusions about what the phenomena describes.</p>	<p>7. How can we find the angle if we know the value of a trig function?</p>	
<p><b>Unit 6 Applications of Trig</b></p> <p>F-TF Prove and apply trigonometric identities</p> <p>8. Prove the Pythagorean identity <math>\sin^2(\theta) + \cos^2(\theta) = 1</math> and use it to calculate trigonometric ratios.</p> <p>9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.</p>	<p>1. Students will be able to derive the pythagorean identity and its alternative forms.</p> <p>2. Students will be able to use the pythagorean identity to reduce the complexity of trigonometric models.</p> <p>3. Students will be able to expand an addition or subtraction model of a trig equation and the double angle formula.</p>	<p>1. What sorts of functions can we construct with the pythagorean identity?</p> <p>2. How can we prove the pythagorean identity using what we learned about the unit circle?</p>	<p>See <a href="#">Trig Identites Exam</a> on CD for a sample Pre-AP assessment</p>
<p><b>Unit 7 Vectors and Special Trig Functions</b></p> <p>N-VM Represent and model with vector quantities.</p> <p>1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., <math>v</math>, <math> v </math>, <math>\ v\ </math>, <math>v</math>).</p> <p>2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.</p> <p>3. (+) Solve problems involving velocity and other quantities that can be represented by vectors.</p> <p>Perform operations on vectors.</p> <p>4. (+) Add and subtract vectors.</p>	<p>1. Students will be able to use vectors drawn by hand to represent motion and forces.</p> <p>2. Students will be able to add and subtract vectors both by hand and by use of algebraic formulas.</p> <p>3. Students will be able to find the components of vectors in order to change a graphical solution into an exact algebraic solution.</p> <p>4. Students will be able to interpret in terms of vectors phrases such as “I live 3 times further away than suzy, but in the opposite direction.”</p> <p>5. Students will be able to use the law of sines and law of cosines on non-right triangles in order to find unknown distances and angles.</p> <p>6. Students will be able to identify</p>	<p>1. How can we use vectors to navigate over land, sea, and air?</p> <p>2. What is the meaning of the magnitude of a vector?</p> <p>3. How can we add vectors graphically?</p> <p>4. What are units that commonly get associated with vectors, and what units are commonly associated with scalars?</p> <p>5. Under what circumstances would it be easier to solve a triangulation using the law of sines?</p>	<p><a href="#">Vectors, Law of Sines, and Law of Cosines Exam</a></p> <p><i>(Some excellent applications of mathematics are available at a website from Cornell University: <a href="http://www.math.cornell.edu/~numb3rs/kostyuk/num109.htm">http://www.math.cornell.edu/~numb3rs/kostyuk/num109.htm</a> is one of these, relevant to this unit.)</i></p> <p><i>Precalculus Final Project: <a href="#">Polynomial Predictions: What will your home town look like in 10 years?</a></i></p>

<p>a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.</p> <p>b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.</p> <p>c. Understand vector subtraction <math>v - w</math> as <math>v + (-w)</math>, where <math>-w</math> is the additive inverse of <math>w</math>, with the same magnitude as <math>w</math> and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.</p> <p>5. (+) Multiply a vector by a scalar.</p> <p>a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as <math>c(v_x, v_y) = (cv_x, cv_y)</math>.</p> <p>b. Compute the magnitude of a scalar multiple <math>cv</math> using <math>\ cv\  =  c v</math>. Compute the direction of <math>cv</math> knowing that when <math> c v \neq 0</math>, the direction of <math>cv</math> is either along <math>v</math> (for <math>c &gt; 0</math>) or against <math>v</math> (for <math>c &lt; 0</math>).</p> <p><b>G-SRT</b> Apply trigonometry to general triangles</p> <p>9. (+) Derive the formula <math>A = 1/2 ab \sin(C)</math> for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.</p> <p>10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.</p> <p>11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).</p>	<p>when law of sines is appropriate and when law of cosines is appropriate based on the situation that is given.</p>		
<p><b>Unit 8</b> Matrix Operations</p> <p>A-REI Solve systems of equations</p> <p>5. Prove that, given a system of two equations in two</p>	<ol style="list-style-type: none"> <li>When presented with a system of two equations, students will be able to find the value of the variables.</li> <li>Students will apply their solutions to situations involving supply and</li> </ol>	<ol style="list-style-type: none"> <li>What are some possible uses for solving systems of equations?</li> <li>What is a general solution for determining</li> </ol>	<p><a href="#">Scatterplots and Systems of Two Equations Quiz</a></p> <p><a href="#">Precalculus Final Exam</a></p>

<p>variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.</p> <p>Common Core State StandardS for matHematICS</p> <p>6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.</p> <p>7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line <math>y = -3x</math> and the circle <math>x^2 + y^2 = 3</math>.</p> <p>8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.</p> <p>9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension <math>3 \times 3</math> or greater).</p>	<p>demand.</p> <p>3. Students will be able to solve systems of two equations graphically and numerically.</p> <p>4. Students will be able to solve systems of equations where one of the equations is not linear graphically and algebraically.</p> <p>5. Students will be able to use matrices to solve systems of 3 or more equations using Row – Echelon Form (REF).</p>	<p>the optimal production rate for a supply and demand problem?</p> <p>3. What are the benefits to solving a system of equations graphically or numerically?</p>	
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